FOUR BASIC FUNCTIONS AND GENERALIZED ELEMENT FACTORS CONSIDERING SAW VELOCITY DISPERSION EFFECT

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1. Abstract

A general approach for the description of the electrostatic field in an infinite regular electrode grating is presented. The general approach leads to four basic functions: a potential-basis, a voltage-basis, a dipole-basis and a charge-basis. These four basic functions are four degrees of freedom to approximate an electrostatic field for IDT of different structures. On the other hand, the usage of these four basic functions in the synthesis problem leads to four types of the IDT topology implementation.

Generalized frequency element factors are written for the assumption the SAW waves propagate with different wave numbers within an electrode and within an inter-electrode gap. This generalization explains the SAW spectrum behavior for high harmonics and especially for the SAW devices made up on piezo-film.

2. Introduction

The exact solution for a two-dimensional electrostatic field of a finite SAW inter-digital transducer (IDT) [1,2] was modified for the effective numerical calculation in [3,4]. However, this solution is not suitable for applying the transversal filters theory and the linear programming in the synthesis problem.

In order to use the linear programming for the IDT topology calculation, the frequency response of the IDT was performed as multiplication of an element factor and an array factor [5-16]. Such a definition of the IDT frequency response became possible due to approximation of the electrostatic field of the IDT with the superposition of the basic solutions for an infinite electrode grating. Only two types of the basic solutions are widespread for the electrostatic field description.

They are

- basic function suggested in [10,11,12,13], where only the zeroth electrode is hot with potential of **+1** and others are grounded. We call this basic solution as a potential-basis.
- basic function suggested in [6,7,8,9], where the electrodes enumerated from $-\infty$ to zero are grounded and the electrodes enumerated from 1 to $+\infty$ are hot with the voltage of **+1.** So the voltage on gap between the zeroth and the first electrodes equals **+1**, whereas the voltages on the other gaps are equal to zero. We call this basic solution as a voltage-basis.

The benefits of two other basic functions are shown in this paper.

They are

- basic function presented in this paper, where only the zeroth electrode is charged with **+1** and others keep no charge. We call this basic solution as a charge-basis.
- basic function suggested in [14,15], where the zeroth electrode is charged with **+1**, the first electrode is charged with **–1** and the other electrodes are isolated and keep no charge. So there is only one charged dipole formed by the zeroth and the first electrodes. We call this basic solution as a dipole-basis.

In practice the SAW IDT structure consists of a finite number of electrodes, so the use of the infinite grating models leads to inherent uncompensated electrostatic end-effect [6,8,9,16,17]. The main beneficial property of the usage of the charge-basis or the dipole-basis in comparison with the potential-basis or the voltage-basis is the fact that the electrostatic end-effect is approximated more exactly.

Quasi-static approach is applied for the frequency element factor definition. Generalized frequency element factors are modified, considering that the SAW waves propagate with different wave numbers within an electrode and within an inter-electrode gap. Such generalization of the quasi-static approach allows describing the SAW spectrum behavior for high harmonics and especially for the SAW devices made up on piezo-film.

3. Prologue

The quasi-static approach simplifies the analysis of generalized SAW transducers [6,7]. This approach is based on the assumption that the charge density is dominated by an electrostatic term, defined as the charge density obtained when acoustic wave excitation is ignored. The frequency spectrum of the SAW acoustic potential $\varphi_a(\omega)$ on the left port of the launching transducer is expressed via the Fourier transform $R(\kappa)$ of the charge distribution density $\rho(x)$:

$$\varphi_a(\omega) = j\,\Gamma_s\,R(\kappa) \qquad (1)$$

where

Γ_s is a constant, depending on the piezoelectric material and cut; this factor is a measure of the piezoelectric coupling of the substrate material.

j is an imaginary unit;

ω is a cycle frequency;

κ is a wave number, $\kappa = \omega / V$;

V is an effective SAW velocity;

$R(\kappa)$ is the Fourier transform of the charge distribution density $\rho(x)$.

The quasi-static approach assumes that the $\rho(x)$ is the electrostatic charge distribution density.

If we consider the SAW device consisting of two uniform SAW transducers, one of which operates as a launcher and the other operates as a detector, then the device response is proportional to the multiplication of the characteristic SAW IDT responses $H_1(\omega)H_2(\omega)$. To analyze the devices, it is convenient to define $H_1(\omega)$ and $H_2(\omega)$ so that both of them would be written by the similar way $H(\omega)$ [6]. In our case the $H(\omega)$ is written by using the expression (1) for the charge distribution density, i.e.

$$H(\omega) = \frac{\sqrt{W\omega\Gamma_S}}{U_t} R(\kappa) \qquad (2)$$

where

U_t is a voltage applied to the SAW transducer.

W is a width of the propagated SAW front.

So the problem of the SAW transducer frequency response definition is mainly the electrostatic problem and the functions $\rho(x)$ and $R(\kappa)$ definition problem.

The superposition method for the function $\rho(x)$ definition may be applied for an infinite periodical electrode grating [5-15], i.e.

$$\rho(x) = \sum_n A(n)\, \rho_o(x - np) \qquad (3)$$

where $\rho_o(x)$ is a basic function and $A(n)$ is the n-th weight, associated with the n-th period of the grating, p is a grating period. So the function $R(\kappa)$ is written as:

$$R(\kappa) = R_O(\kappa) \sum_n A(n) \exp(-j\kappa np) \qquad (4)$$

where $R_o(\kappa)$ is an element factor[6], defined as the Fourier transform of the basic charge distribution density $\rho_o(x)$.

Four types of basic functions: $\rho_\varphi(x)$ and $R_\varphi(\kappa)$ for potential-basis, $\rho_u(x)$ and $R_u(\kappa)$ for voltage-basis, $\rho_q(x)$ and $R_q(\kappa)$ for charge-basis, $\rho_\sigma(x)$ and $R_\sigma(\kappa)$ for dipole-basis are presented in this paper. Here and further the subscript indexes φ, u, q, σ are used for potential-basis, voltage-basis, charge- basis, dipole-basis respectively.

Hitherto it was assumed that the SAW wave propagates with the wave-number κ independent of the coordinate x. However the electrode grating assumes that the electrical and mechanical boundary conditions depend on the coordinate x along the direction of the SAW propagation and thus the SAW velocity dispersion effect takes place around the electrode-gap boundaries. The simplest approximation of the SAW wave-number with κ_e within electrode and κ_g within inter-electrode gap suggests itself [18,19]. Let then κ be an effective wave-number and r be a ratio $r = \kappa_e / \kappa_g$. In this case the SAW potential $\varphi_a(\omega)$ is described by the modified spectrum function $R_o(\kappa,r)$ instead of $R_o(\kappa)$. The physical meaning of the function $R_o(\kappa,r)$ is that a modified frequency spectrum of the SAW excited by oscillated basic distributed charge $\rho_o(x)$, therewith the SAW propagates with the wave-number κ_e within electrodes and κ_g within gaps. Such a generalization of the element factor becomes essential for high harmonics, when the difference between κ_g and κ_e is expected to be significant, especially for devices working on piezo-films.

Note, that $R_o(\kappa, r=1) \equiv R_o(\kappa)$.

The four functions: $R_\varphi(\kappa,r)$, $R_u(\kappa,r)$, $R_q(\kappa,r)$, $R_\sigma(\kappa,r)$ are presented in this paper as well.

4. Problem Formalization

4.1. Two-dimensional problem

To formalize the electrostatic problem we admit the following assumptions.

- The infinite periodical sequence of the parallel electrodes is placed in the (X, Z) - plane. The intervals occupied by the electrodes along the X-axis are (a_n,b_n). An electrode's width equals w, $w = b_n - a_n$, and the grating period equals p, $p=b_n-b_{n-1}$ **(Figure-1).**
- The upper half-plane is the vacuum with the dielectric permittivity ε_o and the lower half-plane is a dielectric material with the effective dielectric permittivity ε_p.
- The electrodes are infinitely long in Z-direction, so the electrostatic problem becomes two-dimensional in the (X, Y) - plane.
- The electrodes have zero thickness and zero electrical resistance.
- The electrodes are enumerated from $-\infty$ to $+\infty$.

4.2. Complex potential

Let us consider a pair of potential functions in order to describe the two-dimensional electrostatic field. The first one is a two-dimensional electrostatic potential $\varphi(x,y)$

and the second one is a two-dimensional electrostatic flux function $\sigma(x,y)$.

Let us introduce the complex potential

$$\Phi(x,y) = \varphi(x,y) + j\sigma(x,y) \quad (5)$$

in order to apply the theory of complex functions in the plane (X,Y) for the two-dimensional electrostatic field description. The complex potential $\Phi(x,y)$ must satisfy the Laplace's equation in the upper half-space and in the lower half-space.

The Cauchy's low links the functions $\varphi(x,y)$ and $\sigma(x,y)$, i.e.

$$\begin{cases} E_y(x,y) = \dfrac{\partial\varphi(x,y)}{\partial y} = -\dfrac{\partial\sigma(x,y)}{\partial x} \\ E_x(x,y) = \dfrac{\partial\varphi(x,y)}{\partial x} = \dfrac{\partial\sigma(x,y)}{\partial y} \end{cases} \quad (6)$$

where $E_x(x,y)$ and $E_y(x,y)$ are the components of the two-dimensional electrostatic field intensity. The electrical boundary conditions for the complex potential $\Phi(x,y)$ along the line $y=0$ are

$$E_x(x, y=0) \equiv 0\,,\ x \in (a_n, b_n) \quad (7a)$$
$$E_y(x, y=0) \equiv 0\,,\ x \in (b_n, a_{n+1}) \quad (7b)$$

Let us introduce the designations for the uniform potential on the n-th electrode and for the uniform flux through the n-th gap between the n-th and the (n+1)-st electrodes:

$$\varphi(n) \equiv \varphi(x, y=0),\ x \in (a_n, b_n) \quad (\varphi.8)$$
$$\sigma(n) \equiv \sigma(x, y=0),\ x \in (b_n, a_{n+1}) \quad (\sigma.8)$$

Let us denote an integral charge on the n-th electrode by $q(n)$ and a voltage on the n-th gap between the n-th and the $(n+1)$-st electrodes by $u(n)$.

$$u(n) = \varphi(n+1) - \varphi(n) \quad (u.8)$$

By the Gauss's law the free charge distribution density is equal to the jump of the electrostatic field displacement discontinuity on the metal space. So the integral charge $q(n)$ on the n-th electrode may be expressed as the difference of the electrostatic field flux values between the neighboring gaps.

$$q(n)/(\varepsilon_o+\varepsilon_p) = \sigma(n) - \sigma(n-1) \quad (q.8)$$

4.3. Potential-basis formalization

By setting the following distribution of the potentials $\varphi(n)$

$$\varphi(n) = +1\,,\ n = 0 \quad (\varphi.9a)$$
$$\varphi(n) = 0\,,\ n \neq 0 \quad (\varphi.9b)$$

we build the potential-basis. The boundary conditions $(\varphi.9a)$-$(\varphi.9b)$ have the physical meaning that the zeroth electrode is hot while others are grounded [6,7,8,9,10].

4.4. Voltage-basis formalization

The boundary conditions

$$u(n) = +1\,,\ n = 0 \quad (u.9a)$$
$$u(n) = 0\,,\ n \neq 0 \quad (u.9b)$$

[6,8,9] give the voltage-basis and have the physical meaning that the electrodes enumerated from $-\infty$ to zero are grounded and the electrodes enumerated from 1 to $+\infty$ are hot with potential of **+1 .**

4.5. Dipole-basis formalization

By assigning the following distribution of the imaginary part of the complex potential

$$\sigma(n) = +1\,,\ n = 0 \quad (\sigma.9a)$$
$$\sigma(n) = 0\,,\ n \neq 0 \quad (\sigma.9b)$$

we build the dipole-basis boundary conditions. The boundary conditions $(\sigma.9a)$-$(\sigma.9b)$ have the physical meaning that all the electrodes excluding both the zeroth and the first electrodes are isolated and keep no charge. The electrostatic voltage applied between the zeroth and the first electrodes is such that the zeroth one is charged with the positive value $+1/(\varepsilon_o+\varepsilon_p)$ and the first one is charged with the negative value $-1/(\varepsilon_o+\varepsilon_p)$ [14,15].

4.6. Charge-basis formalization

Let us assume that we have succeeded to charge the zeroth electrode with the free charge of $(\varepsilon_o+\varepsilon_p)$ while all the other electrodes keep zero charge, i.e.

$$q(n)/(\varepsilon_o+\varepsilon_p) = +1\,,\ n = 0 \quad (q.9a)$$
$$q(n) = 0\,,\ n \neq 0 \quad (q.9b)$$

Such a structure we call as charge-basis.

5. Electrostatic Solution

5.1. Solution for Potential-basis

The charge distribution density for the potential-basis is symmetrical around the zeroth electrode center. It is suitable to choose the point $x=0$ at the zeroth electrode center. The charge distribution density $\rho_\varphi(x)$ satisfying the potential-basis boundary conditions $(\varphi.9a)$-$(\varphi.9b)$ is written by the following equation [6,10,11,12,13]:

$$\rho_\varphi(x) = 0, \quad x \in (b_m\,,\ a_{m+1}) \quad (\varphi.10a)$$

$$\rho_\varphi(x) = \frac{(-1)^m}{p}\frac{(\varepsilon_o+\varepsilon_p)\sqrt{2}\ G_\varphi(x,\theta)}{\sqrt{Cos\frac{2\pi\ x}{p} - Cos\,\theta}}\ ,$$

$$x \in (a_m, b_m) \quad (\varphi.10b)$$

where:

m - is electrode number,
$\theta = \pi\eta$,
η is a metallization ratio, $\eta = w/p$,
w is an electrode width ,
p is a grating period,

$$G_\varphi(x,\theta) = \int_{-1}^{+1} \frac{Cos\frac{\pi\ t}{2}\ Cos\frac{\pi\ x\ t}{p}}{P_{\frac{t-1}{2}}(-Cos\theta)}\,dt$$

$P_s(x)$ - is the Legendre function.

The free charge ***q(m)***, induced on the m-th electrode in the potential-basis is written as:

$$\frac{q_\varphi(m)}{(\varepsilon_o+\varepsilon_p)} = 2\int_0^1 \frac{Sin(\pi\ s)\,Cos(2\pi\ ms)\,P_{-s}(Cos\,\theta)}{P_{-s}(-Cos\,\theta)}\,ds$$

(φ.11a)

The integral in (φ.11a) is simplified for metallization ratio, $\eta = 0.5$:

$$q_\varphi(m) = \frac{4(\varepsilon_o+\varepsilon_p)}{\pi(1-4m^2)} \qquad (\varphi.11b)$$

The value $\sigma_\varphi(m)$ of the electrostatic field flux through the gap between ***m***-th and ***(m+1)***-st electrodes in the potential-basis is expressed as:

$$\sigma_\varphi(m) = \int_0^1 \frac{P_{-s}(Cos\,\theta)}{P_{-s}(-Cos\,\theta)}\,Sin[(2m+1)\pi s]\,ds$$

(φ.12a)

After a number of simplifications for the ratio $\eta=0.5$, it is written as:

$$\sigma_\varphi(m) = \frac{1}{\pi(m+0.5)} \qquad (\varphi.12b)$$

The potential-basis element factor $R_\varphi(\kappa)$, defined as the Fourier transform of the charge distribution density $\rho_\varphi(x)$, is equal to [6,10,11,12]

$$R_\varphi(\kappa) = (\varepsilon_o+\varepsilon_p)\frac{2Sin(\pi s)}{P_{-s}(-Cos\theta)}P_n\,(Cos\theta)$$

$$\text{for} \quad n \le \frac{\kappa\ p}{2\pi} \le n+1, \qquad (\varphi.13)$$

where

$$s = \frac{\kappa\ p}{2\pi} - n, \quad 0 \le s \le 1$$

The usage of the potential-basis leads to interpretation of the $R_o(\kappa)$ in (4) as $R_\varphi(\kappa)$ and the weights ***A(n)*** get meaning of applied potentials to electrodes.

The frequency spectrum $R_\varphi(\kappa,r)$, that is considered the SAW propagates with the wave number κ_e within electrodes and κ_g within gaps, is defined as:

$$\frac{R_\varphi(\kappa,r)}{(\varepsilon_o+\varepsilon_p)} = \frac{2(-1)^n\,Sin(\pi\ s)}{P_{-s}(-Cos\,\theta)}P_{n+\gamma}(Cos\,\theta)$$

$$\text{for} \quad n \le \frac{\kappa\ p}{2\pi} \le n+1, \qquad (\varphi.14)$$

where $\kappa = \eta\kappa_e + (1-\eta)\kappa_g$

$$\gamma = \frac{(r\text{-}1)(1\text{-}\eta)}{1+\eta(r\text{-}1)}\ , \quad r = \frac{\kappa_g}{\kappa_e}$$

$$s = \frac{\kappa\ p}{2\pi} - n, \quad 0 \le s \le 1$$

5.2. Solution for Voltage-basis

The charge distribution density for the voltage-basis is anti-symmetrical around the zeroth gap center $x=p/2$. It is suitable to choose the shifted coordinate $\xi = x-p/2$, so the point $\xi=0$ lies at the zeroth gap center. The charge distribution density $\rho_u(\xi)$ satisfying the voltage-basis boundary conditions (***u.***9a)-(***u.***9b) is written by the following equation:

$$\rho_u(\xi)=0, \quad \xi=x-p/2, \quad x\in(b_m, a_{m+1}) \quad (u.10a)$$

$$\rho_u(\xi) = \frac{(-1)^m}{p}\frac{(\varepsilon_o+\varepsilon_p)\sqrt{2}\ G_u(\xi,\theta)}{\sqrt{Cos(\pi-\theta) - Cos\frac{2\pi\ \xi}{p}}},$$

$$\xi = x - p/2, \quad x \in (a_m, b_m) \qquad (u.10b)$$

where

$$G_u(\xi,\theta) = -\frac{1}{2}\int_{-1}^{+1}\frac{Cos\frac{\pi\,\xi\,t}{p}}{P_{\frac{t-1}{2}}(-Cos\theta)}\,dt$$

The solution $\rho_\varphi(x)$ (φ.10a)-(φ.10b) for the charge distribution of the potential-basis may be considered as superposition of the voltage-basis solutions:

$$\rho_\varphi(x) = \rho_u(\xi-p) - \rho_u(\xi)\,,\ \xi = x-p/2 \qquad (u.10c)$$

And conversely, the solution $\rho_u(\xi)$ (u.10a)-(u.10b) for the charge distribution of the voltage-basis may be considered as superposition of the potentials-basis solutions:

$$\rho_u(\xi) = \sum_{n=1}^{\infty}\rho_\varphi(x - np) \qquad (u.10d)$$

The free charge $q_u(m)$, induced on the m-th electrode in the voltage-basis, is written as:

$$\frac{q_u(m)}{(\varepsilon_o+\varepsilon_p)} = \int_{-1}^{1}\frac{Sin[(2m-1)\pi\,s]\;\;P_{-s}(Cos\,\theta)}{P_{-s}(-Cos\,\theta)}\,ds \qquad (u.11a)$$

The integral in (u.11a) is simplified for the metallization ratio, $\eta = 0.5$:

$$q_u(m) = \frac{(\varepsilon_o+\varepsilon_p)}{\pi(m-0.5)} \qquad (u.11b)$$

The value $\sigma_u(m)$ of the electrostatic field flux through the gap between m-th and $(m+1)$-st electrodes in the voltage-basis, that is defined according to the Gauss's low

$$\sigma_u(m) = \sum_{n=-\infty}^{m} q_u(n) \qquad (u.12)\,,$$

losses a physical meaning in this case as well as in the case of the hot electrodes with numbers $+\infty$. So, the voltage-basis is the non-physical meaning abstraction. Let us remind that the superposition of two voltage-basis functions (u.10c) has the physical meaning of the potential-basis function.

The voltage-basis element factor $R_u(\kappa)$, defined as the Fourier transform of the charge distribution density $\rho_u(x)$, is equal to [6]:

$$\frac{R_u(\kappa)}{(\varepsilon_o+\varepsilon_p)} = \frac{-jR_\varphi(\kappa)}{2Sin(\pi s)} = j\frac{(-1)^{n+1}P_n(Cos\theta)}{P_{-s}(-Cos\theta)} \qquad (u.13)$$

$$\text{for} \quad n \le \frac{\kappa\,p}{2\pi} \le n+1,$$

where

$$s = \frac{\kappa\,p}{2\pi} - n, \quad 0 \le s \le 1$$

The usage of the voltage-basis leads to interpretation of the $R_o(\kappa)$ in (4) as $R_u(\kappa)$ and weight $A(n)$ gets meaning of applied voltages between the (n+1)-st electrode and the n-th one. For apodised structures this weight is proportional to the n-th inter-electrode overlap. This fact does the voltage-basis as well as the potential-basis suitable to analyze and to implement an IDT of uniform and apodised topology, when the end-effect is not essential.

The frequency spectrum $R_u(\kappa,r)$, that is considered the SAW propagates with the wave number κ_e within electrodes and κ_g within gaps, is defined as:

$$\frac{R_u(\kappa,r)}{(\varepsilon_o+\varepsilon_p)} = j\frac{(-1)^{n+1}P_{n+\gamma}(Cos\theta)}{P_{-s}(-Cos\theta)} \qquad (u.14)$$

$$\text{for} \quad n \le \frac{\kappa\,p}{2\pi} \le n+1,$$

where $\kappa = \eta\kappa_e + (1-\eta)\kappa_g$

$$\gamma = \frac{(r\text{-}1)(1\text{-}\eta)}{1+\eta(r\text{-}1)}\ ,\quad r = \frac{\kappa_g}{\kappa_e}$$

$$s = \frac{\kappa\,p}{2\pi} - n, \quad 0 \le s \le 1$$

5.3. Solution for Dipole-basis

The charge distribution density for the dipole-basis is symmetrical around the zeroth gap center $x=p/2$. So as well as in the voltage-basis case, it is suitable to choose shifted coordinate $\xi = x-p/2$, so the point $\xi=0$ lies at the zeroth gap center. The charge distribution density $\rho_\sigma(\xi)$, satisfying the dipole-basis boundary conditions (σ.9a)-(σ.9b), is written by the following equation [14,15]:

$$\rho_\sigma(\xi)=0,\ \ \xi=x-p/2,\ \ x\in(b_m,\,a_{m+1}) \qquad (\sigma.10a)$$

$$\rho_\sigma(\xi) = \frac{(-1)^m}{p} \frac{(\varepsilon_o + \varepsilon_p)\sqrt{2}\ G_\sigma(\xi,\theta)}{\sqrt{Cos(\pi - \theta) - Cos\dfrac{2\pi\xi}{p}}}\ ,$$

$$\xi = x - p/2\ ,\quad x \in (a_m, b_m) \qquad (\sigma.10b)$$

where

$$G_\sigma(\xi,\theta) = \int_{-1}^{+1} \frac{Cos\dfrac{\pi t}{2}\, Cos\dfrac{\pi\xi t}{p}}{P_{\frac{t-1}{2}}(Cos\theta)}\, dt$$

The voltage $u_\sigma(m)$ in the gap between the m-th and the (m+1)-st electrodes in the dipole-basis structure is:

$$u_\sigma(m) = -2\int_0^1 \frac{Sin(\pi s)Cos(2\pi m s)P_{-s}(-Cos\theta)}{P_{-s}(Cos\theta)}\, ds \qquad (\sigma.11a)$$

The integral in (σ.11a) may be simplified for the metallization ratio η = 0.5, i.e.

$$u_\sigma(m) = \frac{4}{\pi(4m^2 - 1)} \qquad (\sigma.11b)$$

The associated potential $\varphi_\sigma(m)$ on the m-th electrode in the dipole-basis structure is equal to:

$$\varphi_\sigma(m) = -\int_0^1 \frac{P_{-S}(-Cos\,\theta)}{P_{-S}(Cos\,\theta)} Sin[(2m-1)\pi s]\, ds \qquad (\sigma.12a)$$

The integral in (σ.12a) may be simplified for the metallization ratio η = 0.5, i.e.

$$\varphi_\sigma(m) = -\frac{1}{\pi\,(m - 0.5)} \qquad (\sigma.12b)$$

The dipole-basis element factor $R_\sigma(\kappa)$, defined as the Fourier transform of the charge distribution density $\rho_\sigma(x)$, is equal to [14,15]

$$\frac{R_\sigma(\kappa)}{(\varepsilon_o + \varepsilon_p)} = j\frac{2Sin(\pi\, s)}{P_{-s}(\,Cos\theta\,)} P_n\,(Cos\theta\,)$$

$$\text{for} \quad n \le \frac{\kappa\, p}{2\pi} \le n+1, \qquad (\sigma.13)$$

where

$$s = \frac{\kappa\, p}{2\pi} - n, \quad 0 \le s \le 1$$

The usage of the dipole-basis leads to interpretation of the $R_o(\kappa)$ in (4) as the $R_\sigma(\kappa)$ and the weights $A(n)$ get meaning of electrostatic field flux values through the inter-electrode gaps. An absence of charges outside of IDT is described with zero electrostatic flux. So the electrostatic end-effect is approximated by $A(n)$ = **0**, for all numbers n lying out of the IDT inter-electrode gap numbers. This fact advantageously distinguishes the dipole-basis from the both potential-basis and voltage-basis with accurate approximation of the electrostatic end-effect.

The frequency spectrum $R_\sigma(\kappa,r)$, that is considered the SAW propagates with the wave number κ_e within electrodes and κ_g within gaps, is defined as:

$$\frac{R_\sigma(\kappa,r)}{(\varepsilon_o + \varepsilon_p)} = j\frac{2Sin(\pi\, s)}{P_{-s}(\,Cos\theta\,)} P_{n+\gamma}(Cos\theta\,) \qquad (\sigma.14)$$

$$\text{for} \quad n \le \frac{\kappa\, p}{2\pi} \le n+1,$$

where $\kappa = \eta\kappa_e + (1-\eta)\kappa_g$

$$\gamma = \frac{(r\text{-}1)(1\text{-}\eta)}{1+\eta(r\text{-}1)}\ ,\quad r = \frac{\kappa_g}{\kappa_e}$$

$$s = \frac{\kappa\, p}{2\pi} - n, \quad 0 \le s \le 1$$

5.4. Solution for Charge-basis

The charge distribution density for the charge-basis is symmetrical around the zeroth electrode center. It is suitable to choose the point $x=0$ at the zeroth electrode center. The charge distribution density $\rho_q(x)$ satisfying the charge-basis boundary conditions (q.9a)-(q.9b), saying that the zeroth electrode has free charge equal $(\varepsilon_o+\varepsilon_p)$ while all the other electrodes keep zero charge, is written by the following equation:

$$\rho_q(x) = 0, \qquad x \in (b_m, a_{m+1}) \qquad (q.10a)$$

$$\frac{\rho_q(x)}{(\varepsilon_o + \varepsilon_p)} = \frac{(-1)^m}{p} \frac{\sqrt{2}\ G_q(x,\theta)}{\sqrt{Cos\frac{2\pi x}{p} - Cos\theta}},$$

$$x \in (a_m, b_m) \qquad (q.10b)$$

where

$$G_q(x,\theta) = \frac{1}{2}\int_{-1}^{+1} \frac{Cos\frac{\pi x t}{p}}{P_{\frac{t-1}{2}}(Cos\theta)}\, dt$$

The solution $\rho_\sigma(\xi)$ (σ.10a)-(σ.10b) for the charge distribution of the dipole-basis may be considered as the superposition of the charge-basis solutions:

$$\rho_\sigma(\xi) = \rho_q(x) - \rho_q(x-p), \quad \xi = x - p/2 \quad (q.10c)$$

And conversely, the solution $\rho_q(x)$ (q.10a)-(q.10b) for the charge distribution of the charge-basis may be considered as the superposition of the dipole-basis solutions:

$$\rho_q(x) = \sum_{n=0}^{\infty} \rho_\sigma(\xi - np) \qquad (q.10d)$$

The inter-electrode voltages $u_q(m)$ on the m-th gap in the charge-basis is written as:

$$u_q(m) = \int_{-1}^{+1} \frac{Sin[(2m-1)\pi s]\ \ P_{-s}(-Cos\theta)}{P_{-s}(Cos\theta)}\, ds \quad (q.11a)$$

The integral in (q.11a) is simplified for metallization ratio, $\eta = 0.5$:

$$u_q(m) = \frac{(-1)}{\pi(0.5+m)} \qquad (q.11b)$$

The associated potential $\varphi_q(m)$ on the m-th electrode in the charge-basis structure, which is defined as sum of voltages

$$\varphi_q(m) = \sum_{n=-\infty}^{m} u_q(n) \qquad (q.12),$$

losses a physical meaning in this case as well as the not-zero sum charge of the system. So, the charge-basis is the non-physical meaning abstraction. Let us remind that the superposition of two charge-basis functions (q.10c) has the physical meaning of the dipole-basis function.

The charge-basis element factor $R_q(\kappa)$ defined as the Fourier transform of the charge distribution density $\rho_q(x)$ is equal to

$$\frac{R_q(\kappa)}{(\varepsilon_o + \varepsilon_p)} = \frac{jR_\sigma(\kappa)}{2Sin(\pi s)} = \frac{P_n(Cos\theta)}{P_{-s}(Cos\theta)} \qquad (q.13)$$

$$\text{for} \quad n \le \frac{\kappa p}{2\pi} \le n+1,$$

where

$$s = \frac{\kappa p}{2\pi} - n, \quad 0 \le s \le 1$$

The use of the charge-basis leads to interpretation of the $R_o(\kappa)$ in (4) as the $R_q(\kappa)$ and the weights $A(n)$ get meaning of charges induced on electrodes. An absence of charges outside of IDT is described with zero charges on electrodes. So the electrostatic end-effect is approximated by $A(n) = 0$, for all numbers n out of the IDT electrodes numbers. So the charge-basis as well as dipole-basis advantageously distinguish from the both potential-basis and voltage-basis with accurate approximation of the electrostatic end-effect.

The frequency spectrum $R_q(\kappa,r)$, that is considered the SAW propagates with the wave number κ_e within electrodes and κ_g within gaps, is defined as:

$$\frac{R_q(\kappa,r)}{(\varepsilon_o + \varepsilon_p)} = \frac{P_{n+\gamma}(Cos\theta)}{P_{-s}(Cos\theta)} \qquad (q.14)$$

$$\text{for} \quad n \le \frac{\kappa p}{2\pi} \le n+1,$$

where $\kappa = \eta\kappa_e + (1-\eta)\kappa_g$

$$\gamma = \frac{(r\text{-}1)(1\text{-}\eta)}{1+\eta(r\text{-}1)}, \quad r = \frac{\kappa_g}{\kappa_e}$$

$$s = \frac{\kappa p}{2\pi} - n, \quad 0 \le s \le 1$$

6. Calculations

Calculations of the charge distribution for the four basic structures are presented in the Figures-2,3. The comparison of the end-effects does the usage of the dipole-basis be the most preferable.

The element factor and the modified frequency spectrum of SAW for the dipole-basis are presented in the Figure-4. The function $R_\sigma(\kappa,r)$ was calculated for the metallization ratio $\eta=0.5$ with assumption the parameter $r=1.1$. This calculation shows the difference between the functions $R_\sigma(\kappa)$ and $R_\sigma(\kappa,r)$ in the high harmonics range.

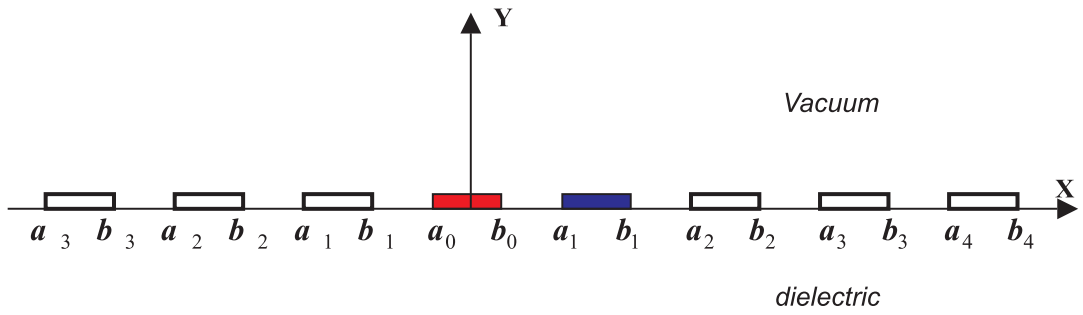


Figure-1.
The intervals (a_n,b_n) along X-axis are occupied by electrodes.
The electrode's width equals w, $w= b_n-a_n$, and the grating period equals p, $p= b_n-b_{n-1}$.

Figure-2.
Charge Distribution in Basic Structures
Potential-Basis
Charge-Basis

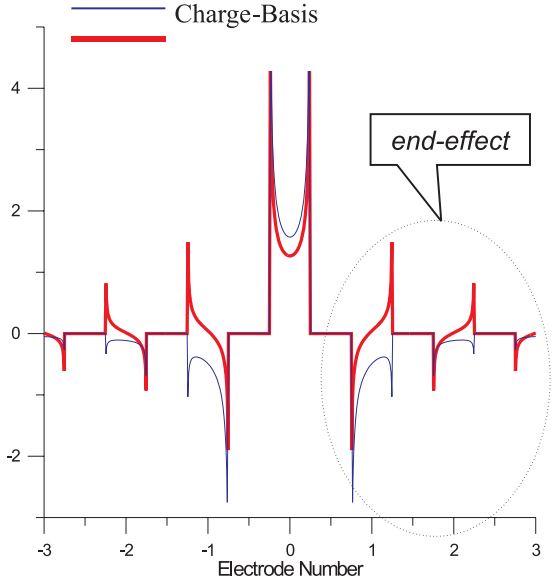
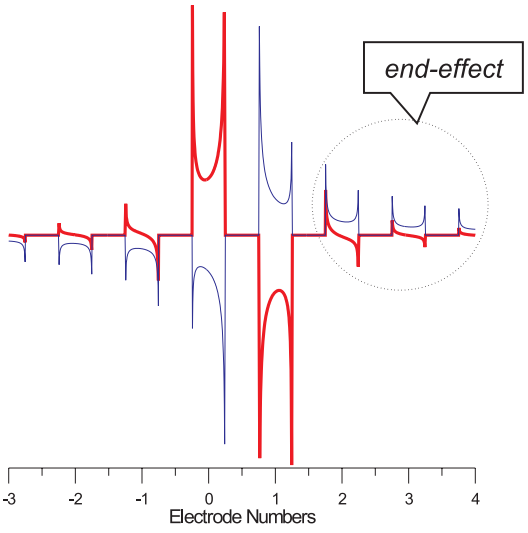


Figure-3.
Charge Distribution in Basic Structures
Voltage-Basis
Dipole-Basis



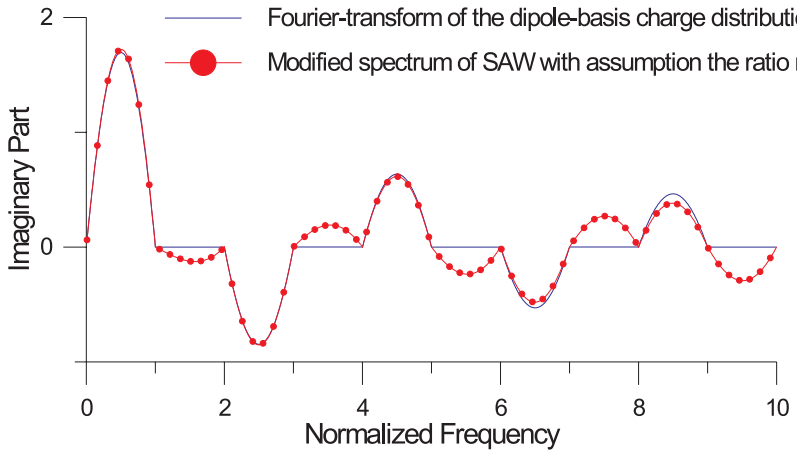


Figure-4. Dipole-basis element factor for metallization ratio

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